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Mathematical Modelling on the Medication for the Growth of Diabetic Cases

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Abstract: Diabetes mellitus is fast growing serious disease in human population. The population of diabetesboth minor and major cases have been discussed through mathematical model. The mathematical model is used to describe the various stages of diabetes with minor and major complications, which is solved by Runge-Kutta method.The model provides the status of the complications of diabetes for medication.

Keywords: Diabetes, Runge-Kutta, Population, Medication

Introduction: Diabetes is a disorder of metabolism in the body. This disorder is generally due to a combination of hereditary and environmental causes. In 2013, WHO developed plans for prevention and control of non-communicable diseases by 2025, which included a 25% reduction in mortality from non-communicable diseases, halting the growth in diabetes and obesity and ensuring that at least 80% of patients have access to affordable basic technologies and essential medicines for non-communicable diseases. According to the burden of diabetes determines for the 1.3 billion population of India mask wide variations across the states of the country, many of which are comparable to large countries in terms of population. The India state-level disease burden initiative has recently reported the overall trends of the diseases, and risk factors from 1990 to 2016 for every state of India. Results are reported for 31 geographical units in India that is 29 states, union territory of Delhi, Chandigarh and others. In this work the numerical data has been taken from Chandigarh region for the research on the disease. Anusha S. et al. 2021 worked on the mathematical modelling co-existence of diabetes and CoV-19 for finding the eco-epidemiological perspective and the burden of the

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disease. In WHO reports, epidemiological analysis are focusing on the co-infection of COVID-19 and comorbidities, because comorbidities have been recognized as a risk factor for increased COVID-19 cases and higher fatality. There are many studies have provided that diabetes is one of the major co-morbidities associated with severe illness and death from COVID-19 infection. Here, the main focus on the dynamics of the co-infection of diabetes and COVID-19 and achieved the following points:

I. To construct the deterministic model that describes diabetes and COVID-19 co-infection with the study of existence of equilibrium and stability.

II. Stochastic models are more realistic as compared to deterministic models. So, the proposed model is extended to a stochastic model to examine the effects of stochastic perturbations.

The approximation solution of the differential equation $\frac{dy}{dx} = f(x,y)$

with the condition $y(x_0) = y_0$

Compute, $x_1 = x_0 + h$

$K_1 = f(x_0, y_0)$

$K_2 = f(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} K_1)$

$K_3 = f(x_0 + h, y_0 - h\, K_1 + 2hK_2)$

$y_1 = y_0 + \frac{1}{6}(K_1 + 4K_2 + K_3)$

Then, for the r input vector, $\bar{x}$ of length; n the Jacobian vector of size nx1 can be defined as

$$J = \frac{df(x)}{dx} = \left[\frac{\partial f(x)}{\partial x_1}, \dots \dots \frac{\partial f(x)}{\partial x_n}\right]$$

So, for the same input vector n, of the length; u the Jacobian is now a $m \times n$ matrix; So, $J \in R^{m \times n}$ that is defined by

$$J = \frac{df(x)}{dx} = \left[\frac{\partial f(x)}{\partial x_1}, \dots \dots \frac{\partial f(x)}{\partial x_n}\right] = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \dots & \frac{\partial f_1(x)}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_m(x)}{\partial x_1} & \dots & \frac{\partial f_m(x)}{\partial x_n} \end{bmatrix}$$

Generalization of system of n equations

$y_1 = f_1(x_1, x_2, \dots x_n)$

$\vdots$

$y_n = f_n(x_1, x_2, \ldots x_n)$

Then, the Jacobian matrix is

$$J = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & \cdots & \frac{\partial y_n}{\partial x_n} \end{bmatrix}$$

for example, to determine the behaviour of general 2D systems of differential equations

$$\frac{\mathrm{dx}}{\mathrm{dt}} = f(x,y)$$

$$\frac{\mathrm{dy}}{\mathrm{dt}} = g(x,y)$$

Which together constitute the matrix defining the linearized system

$$J = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

This Jacobian matrix is called the Jacobian of the given system in the point $(\bar{x}, \bar{y})$.

Formulation of Mathematical Model: The equation of the diabetes is given as below:

$$\frac{dX}{dt} = \Lambda - (\beta_1 + \beta_2)X - \mu X \qquad (1)$$

$$\frac{dY_1}{dt} = \beta_1 X - \mu Y_1 - \eta_1 Y_1 \qquad (2)$$

$$\frac{dY_2}{dt} = \beta_2 X - \mu Y_2 - \eta_2 Y_2 \qquad (3)$$

Where,

A. *Nomenclature:*

B. *$X(t)$: Number of diabetic patients without complications*

C. *$Y_1(t)$: Number of diabetic patients with minor complications*

D. *$Y_2(t)$: Number of diabetic patients with major complications*

E. *N: Total populations (in Millions)*

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F. Λ: *Incidence of diabetes*

G. β_1: *Growth rate of diabetic patientswith minor complications*

H. β_2: *Growth rate of diabetic patients with major complications*

I. μ: *Death rate*

J. η_1: *Death due to minor complications*

K. η_2: *Death due to major complications*

L. *t: time*

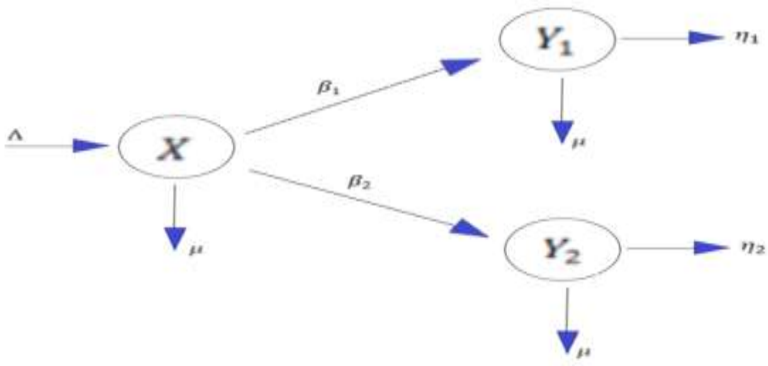

Figure 1: Mathematical Model XY_1Y_2

Analysis of Mathematical Model:

Method :1Runge-Kutta Method

Also, rewrite the above equations

$\frac{dX}{dt} = f_1 = \Lambda - (\beta_1 + \beta_2 + \mu)X; \frac{dY_1}{dt} = f_2 = \beta_1 x - (\eta_1 + \mu)Y_1$

$\frac{dY_2}{dt} = f_3 = \beta_2 x - (\eta_2 + \mu)Y_2$

Using RungeKutta method of order third.

$X_{n+1} = X_n + \frac{1}{6} h \, [K_1 + 4K_2 + K_3].$

Here, $K_1 = f(x_n, y_n); K_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2} K_1); K_3 = f(x_n + h, y_n - hK_1 + 2hK_2)$

So, the general equations for r $\geq$ 0 and h equals totime interval are given by,

$X_{r+1} = X_r + \mathrm{K} = X_r + \frac{1}{6} [K_1 + 4K_2 + K_3]$ (4)

$(Y_1)_{r+1} = (Y_1)_r + \mathrm{L} = (Y_1)_r + \frac{1}{6} [L_1 + 4L_2 + L_3]$ (5)

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$$(Y_2)_{r+1} = (Y_2)_r + \mathrm{M} = (Y_2)_r + \frac{1}{6}[M_1 + 4M_2 + M_3] \qquad (6)$$

Here K, L, and M are defined as

$$K_1 = \Lambda h - (\beta_1 + \beta_2 + \mu)hX_0$$

$$K_2 = \Lambda h - (\beta_1 + \beta_2 + \mu)h\,(X_0 + \frac{1}{2}h\,K_1)$$

$$K_3 = \Lambda h - (\beta_1 + \beta_2 + \mu)h\,(X_0 + h\,K_2)$$

Then, $K = \frac{1}{6}[K_1 + 4K_2 + K_3]$.

$$L_1 = h\beta_1 X_0 - h(\eta_1 + \mu)(Y_1)_0$$

$$L_2 = h\beta_1\left(X_0 + \frac{h}{2}K_1\right) - h(\eta_1 + \mu)\{(Y_1)_0 + hL_1\}$$

$$L_3 = h\beta_1\left(X_0 + \frac{h}{2}K_2\right) - h(\eta_1 + \mu)\{(Y_1)_0 + hL_2\}$$

Then, $L = \frac{1}{6}[L_1 + 4L_2 + L_3]$.

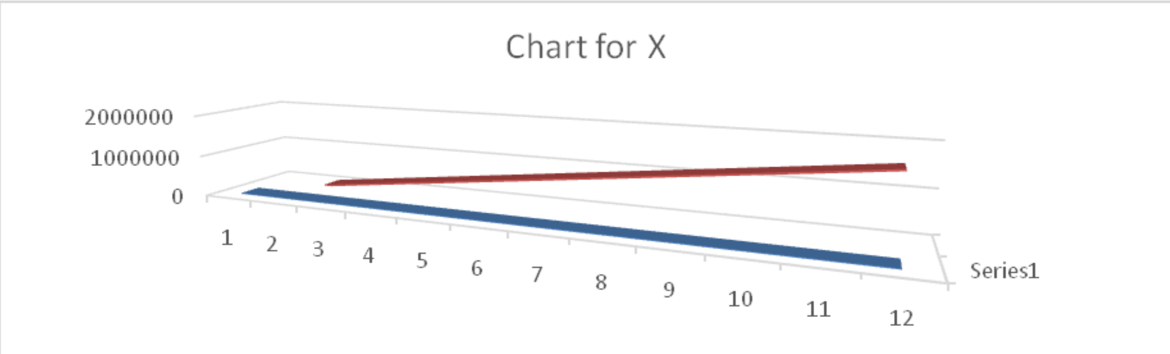
$$M_1 = h\beta_2 X_0 - h(\eta_2 + \mu)(Y_2)_0$$

$$M_2 = h\beta_2\left(X_0 + \frac{h}{2}K_1\right) - h(\eta_2 + \mu)\left\{(Y_2)_0 + \frac{1}{2}hM_1\right\}$$

$$M_3 = h\beta_2\left(X_0 + \frac{h}{2}K_2\right) - h(\eta_2 + \mu)\{(Y_2)_0 + hM_2\}$$

Then, $M = \frac{1}{6}[M_1 + 4M_2 + M_3]$.

In the report, provided by the ICMR that the prevalence of diabetes in Chandigarh was 13%, 20% of prediabetics and 50% don't know that they have diabetes unless there is complication with one or the other organ.



	1	2	3	4	5	6	7	8	9	10	11	12
Series1	0	0	1	2	3	4	5	6	7	8	9	10
Series2		1215	155383	309550	463718	617885	772053	926220	108038	123455	138871	154288

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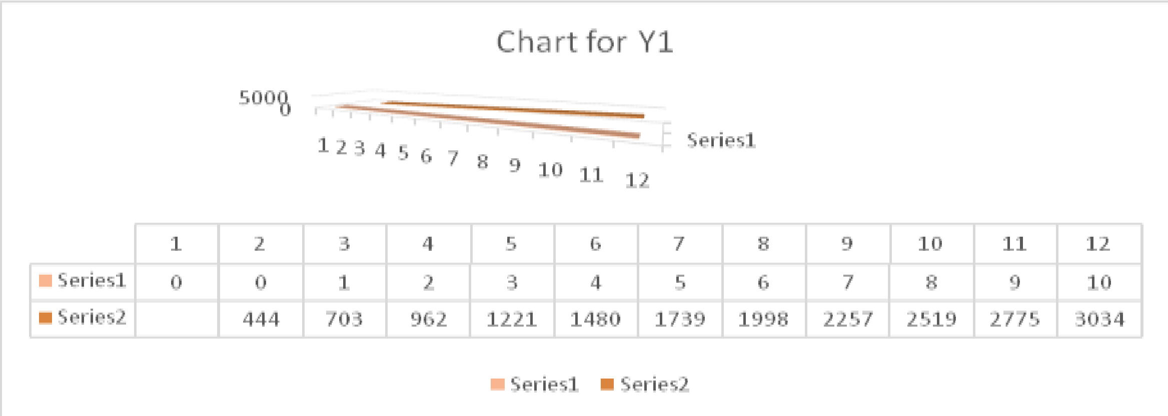
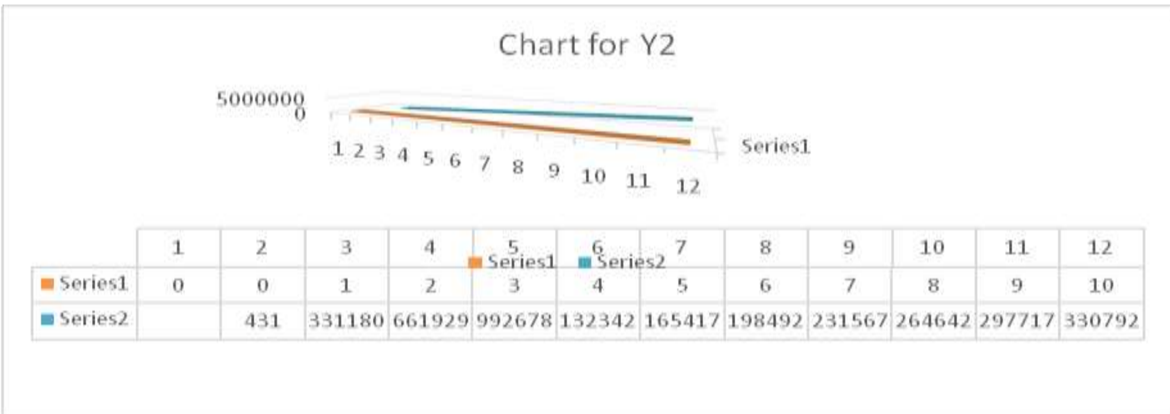
Figure 2: Chart for X(t) that is number of diabetic patients without complications are increases with respect to time.



	1	2	3	4	5	6	7	8	9	10	11	12
Series1	0	0	1	2	3	4	5	6	7	8	9	10
Series2		444	703	962	1221	1480	1739	1998	2257	2519	2775	3034

Figure 3: Chart for $Y_1(t)$ that is number of diabetic patients with minor complications are increases with respect to time.



	1	2	3	4	5	6	7	8	9	10	11	12
Series1	0	0	1	2	3	4	5	6	7	8	9	10
Series2		431	331180	661929	992678	132342	165417	198492	231567	264642	297717	330792

Figure 4: Chart for $Y_2(t)$ that is number of diabetic patients with major complications are increases with respect to time.

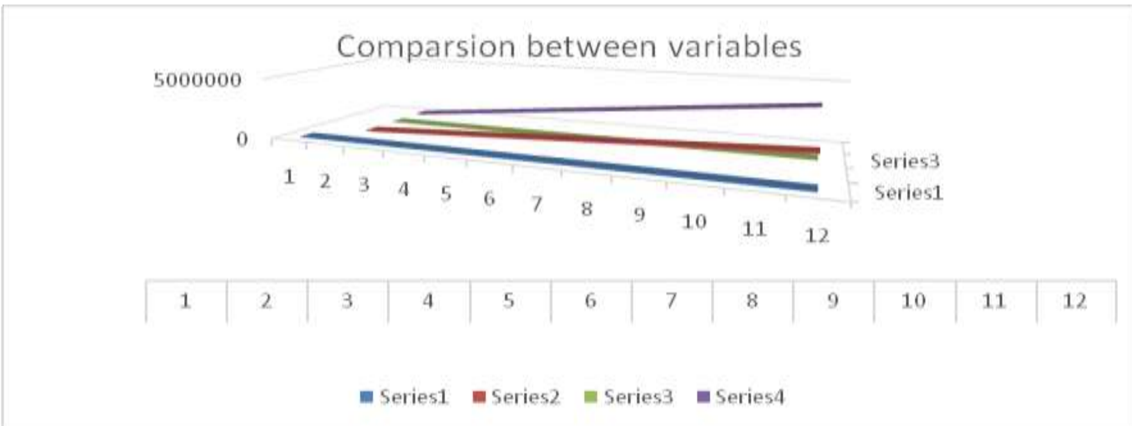


Fig. 5: Comparison of all variablesunder complications is increases with respect to time

Method :2Jacobian Method

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Using the governing equation, construct the Jacobian Matrix (J) as follow,

$$J = \begin{bmatrix} -\beta_1 - \beta_2 - \mu & 0 & 0 \\ \beta_1 & -x_1 - \mu & 0 \\ \beta_2 & 0 & -x_2 - \mu \end{bmatrix}$$

Now the Characteristic equation on Jacobian as given below:

$\det(J - \lambda I) = 0,$

$$\det(J - \lambda I) = \begin{vmatrix} -\beta_1 - \beta_2 - \mu - \lambda & 0 & 0 \\ \beta_1 & -x_1 - \mu - \lambda & 0 \\ \beta_2 & 0 & -x_2 - \mu - \lambda \end{vmatrix} = 0$$

$\Rightarrow (\lambda + \beta_1 + \beta_2 + \mu)(\lambda + x_1 + \mu)(\lambda + x_2 + \mu) = 0$

Therefore, after solving, three eigen values are given as

$\lambda_1 = -(\beta_1 + \beta_2 + \mu);\ \lambda_2 = -(x_1 + \mu);\lambda_3 = -(x_2 + \mu)$

Since, all the eigen values are negative the system is stable. That is under some precautions it will be not increasedor may be minor changing in the growth of diabetic cases.

Graphically Representation of the Outcomes:

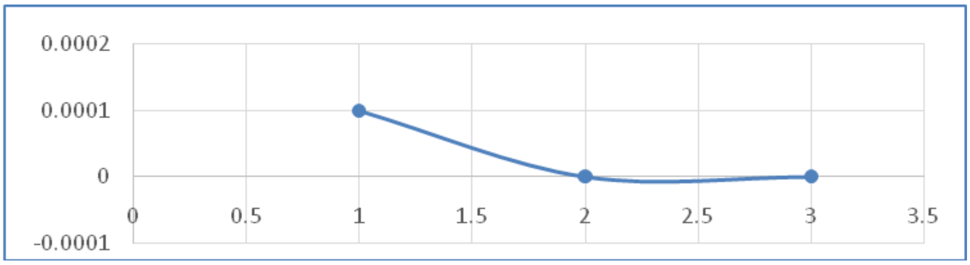

Figure 6: Graph for X and time t.

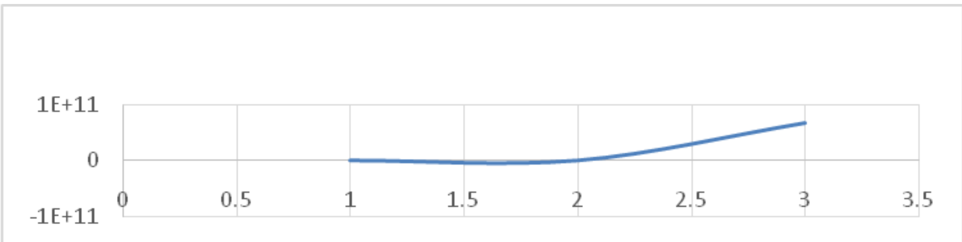

Figure 7: Graph for Y_1 and time t.

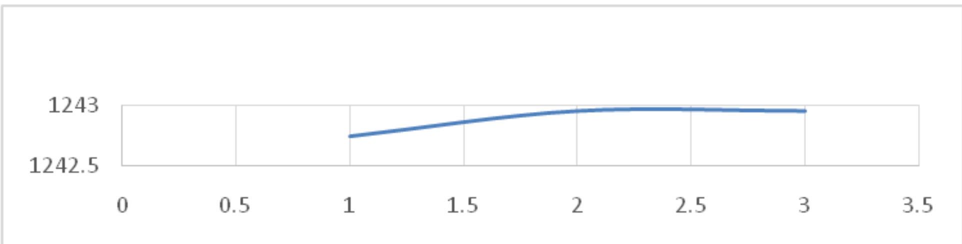

Figure 8: Graph for Y_2 and time t.

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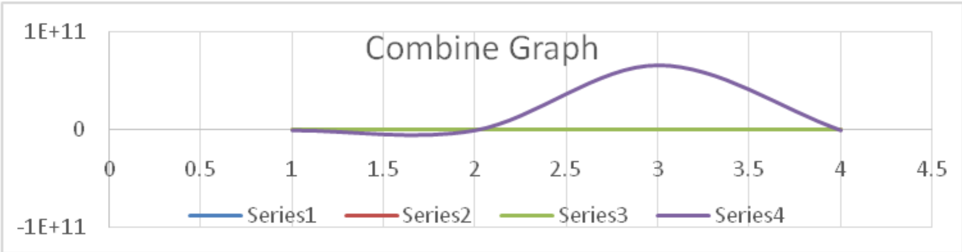

Figure 9: Graph for the all variables with time t.

Discussion: There are plotted four diagrams that model is useful for understanding the growth of diabetic patient with their minor and major complications. The signs of eigen values are found all negative so that the given system will be stableThat is under some precautions it will be not increased or may be minor changing in the growth of diabetic cases. This is important to notice that the doctors and patients alike can have a better understanding of how to medicate along with the minor and major complications during treatment. However, the system maybe unstable in case of not focusing to diabetic patients on time,otherwise heavy less could be there. The mathematical model is used to describe the various stages of diabetes with minor and major complications graphically with the help of Runge-Kutta method and Jacobian method. Comparison of all variables for minor and major complications are analysed graphically with respect to time.The model provides the status of the complications of diabetes for medication through graphically.

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